Free boson representation of $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ at level one

Bo-yu Hou[a]*, Wen-Li Yang[b]† and Yi Zhen[a]‡

[a]Institute of Modern Physics , Northwest University Xian 710069 , P.R.China

[b] Physikalishes Institut der Universität Bonn, Nussalle 12, 53115 Bonn, Germany

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Abstract

We construct a realization of the central extension of super-Yangian double $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ at level-one in terms of free boson fields with a continuous parameter.

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1 Introduction

The algebraic analysis method based on infinite-dimensional highest weight representations of $non-abelian$ symmetries such as deformed infinite-dimensional (super) algebras has proved eminently successful in solving lattice integrable models[1, 2, 3, 4] and completely integrable field theories[5, 6]. A powerful approach for studying the highest weight representations is the bosonization technique[7, 8] which allows one to explicitly construct these objects in terms of the deformed free bosonic fields.

*E-mail address: byhou@phy.nwu.edu.cn

†E-mail address: wlyang@th.physik.uni-bonn.de

‡E-mail address: zheny@phy.nwu.edu.cn

Free bosonic realization of level-one representations has been constructed for most quantum affine algebras[7, 8, 9]. This bosonization technique has also been extended to type I quantum affine super algebras $U_q[\widehat{sl}(M+1|N+1)]$, $M \neq N$[10], $U_q[\widehat{gl}(N|N)]$[11] and $U_q[osp(2|2)^{(2)}]$[12]. On the other hand, free bosonic realizations of Yangian-deformed algebras have been constructed for $DY_\hbar(\widehat{sl}_2)$ at level k[13], $DY_\hbar(\widehat{gl}_N)$ at level one[14] and level k[15], and $A_{\hbar,\eta}(\widehat{sl}_2)$ at level one[16]. It should be remarked that the free bosonic field with continuous parameter initiated by Jimbo et al[17] is powerful in constructing the representations of Yangian-deformed algebras[18, 15, 16]. The purpose of this paper is to construct the central extension super-Yangian double (super-affine-Yangian algebra) $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ algebra at level one in terms of free bosonic fields with continuous parameter.

This paper is arranged as follows. In section 2 we give the defining relations of Drinfield currents of super-affine-Yangian algebra $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$. The free bosonic realization of $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ are constructed in section 3. Appendix contains some detailed calculations.

2 Drinfeld currents of $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$

We will study the super-affine-Yangian algebra $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ for $M, N = 0, 1, \cdots$. The super-affine Yangian algebra $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ is an associative algebra generated by the Drinfeld currents: $X_i^\pm(\mu)$, $\Psi_j^\pm(\mu)$, $i = 1, 2, ...M+N+1, j = 1, 2, ..., M+N$ and a central element c. The generators $\Psi_j^\pm(\mu)$ are invertible. The $\mathbf{Z}_2$-grading of the generators is : $\left[X_{M+1}^\pm(\mu)\right] = 1$ and zero otherwise . The defining relations are [19, 20]

$$\begin{aligned}
\Psi_{i+1}^-(\nu)^{-1}\Psi_i^+(\mu) &= \frac{(\mu-\nu-2\hbar)(\mu-\nu+2\hbar)}{(\mu-\nu)(\mu-\nu)}\Psi_i^+(\mu)\Psi_{i+1}^-(\nu)^{-1}, \quad M+N+1 \geq i \geq M+1,\\
\Psi_{i+1}^-(\nu)^{-1}\Psi_i^+(\mu) &= \frac{(\mu-\nu)(\mu-\nu)}{(\mu-\nu-2\hbar)(\mu-\nu+2\hbar)}\Psi_i^+(\mu)\Psi_{i+1}^-(\nu)^{-1}, \quad i < M+1,\\
\Psi_{i+1}^+(\nu)^{-1}\Psi_i^-(\mu) &= \frac{(\mu-\nu-2\hbar)(\mu-\nu+2\hbar)}{(\mu-\nu)(\mu-\nu)}\Psi_i^-(\mu)\Psi_{i+1}^+(\nu)^{-1}, \quad i < M+1,\\
\Psi_{i+1}^+(\nu)^{-1}\Psi_i^-(\mu) &= \frac{(\mu-\nu)(\mu-\nu)}{(\mu-\nu-2\hbar)(\mu-\nu+2\hbar)}\Psi_i^-(\mu)\Psi_{i+1}^+(\nu)^{-1}, \quad M+N+1 \geq i \geq M+1,\\
\Psi_i^+(\mu)\Psi_i^-(\nu) &= \frac{(\mu-\nu-3\hbar)(\mu-\nu+3\hbar)}{(\mu-\nu+\hbar)(\mu-\nu-\hbar)}\Psi_i^-(\nu)\Psi_i^+(\mu), \quad i < M+1,
\end{aligned}$$

$$
\begin{aligned}
\Psi_i^+(\mu)\,\Psi_i^-(\nu) &= \frac{(\mu-\nu-\hbar)\,(\mu-\nu+\hbar)}{(\mu-\nu+3\hbar)\,(\mu-\nu-3\hbar)}\Psi_i^-(\nu)\,\Psi_i^+(\mu)\,, \qquad M+N+1 \geq i > M+1 \\
\Psi_{M+1}^+(\mu)\,\Psi_{M+1}^-(\nu) &= \Psi_{M+1}^-(\nu)\,\Psi_{M+1}^+(\mu)
\end{aligned}
$$

$$
\begin{aligned}
\Psi_i^+(\mu)^{-1}\,X_i^+(\nu)\,\Psi_i^+(\mu) &= \frac{(\mu_+-\nu-3\hbar)}{(\mu_+-\nu+\hbar)}\,X_i^+(\nu)\,, \qquad M+N+1 \geq i > M+1, \\
\Psi_i^+(\mu)^{-1}\,X_i^+(\nu)\,\Psi_i^+(\mu) &= \frac{(\mu_+-\nu+\hbar)}{(\mu_+-\nu-3\hbar)}\,X_i^+(\nu)\,, \qquad i < M+1, \\
\Psi_i^-(\mu)^{-1}\,X_i^+(\nu)\,\Psi_i^-(\mu) &= \frac{(\mu_--\nu+3\hbar)}{(\mu_--\nu-\hbar)}\,X_i^+(\nu)\,, \qquad i < M+1, \\
\Psi_i^-(\mu)^{-1}\,X_i^+(\nu)\,\Psi_i^-(\mu) &= \frac{(\mu_--\nu-\hbar)}{(\mu_--\nu+3\hbar)}\,X_i^+(\nu)\,, \quad M+N+1 \quad \geq i > M+1, \\
\Psi_{i+1}^+(\mu)^{-1}\,X_i^+(\nu)\,\Psi_{i+1}^+(\mu) &= \frac{(\mu_+-\nu-2\hbar)}{(\mu_+-\nu)}\,X_i^+(\nu)\,, \qquad i < M+1, \\
\Psi_{i+1}^+(\mu)^{-1}\,X_i^+(\nu)\,\Psi_{i+1}^+(\mu) &= \frac{(\mu_+-\nu)}{(\mu_+-\nu-2\hbar)}\,X_i^+(\nu)\,, \qquad M+N+1 \geq i \geq M+1, \\
\Psi_{i+1}^-(\mu)^{-1}\,X_i^+(\nu)\,\Psi_{i+1}^-(\mu) &= \frac{(\mu_--\nu)}{(\mu_--\nu+2\hbar)}\,X_i^+(\nu), \qquad i < M+1, \\
\Psi_{i+1}^-(\mu)^{-1}\,X_i^+(\nu)\,\Psi_{i+1}^-(\mu) &= \frac{(\mu_--\nu+2\hbar)}{(\mu_--\nu)}\,X_i^+(\nu)\,, \qquad M+N+1 \geq i \geq M+1, \\
\Psi_i^+(\mu)^{-1}\,X_i^-(\nu)\,\Psi_i^+(\mu) &= \frac{(\mu_--\nu-\hbar)}{(\mu_--\nu+3\hbar)}\,X_i^-(\nu)\,, \qquad i < M+1, \\
\Psi_i^+(\mu)^{-1}\,X_i^-(\nu)\,\Psi_i^+(\mu) &= \frac{(\mu_--\nu+3\hbar)}{(\mu_--\nu-\hbar)}\,X_i^-(\nu)\,, \qquad M+N+1 \geq i > M+1, \\
\Psi_i^-(\mu)^{-1}\,X_i^-(\nu)\,\Psi_i^-(\mu) &= \frac{(\mu_+-\nu-3\hbar)}{(\mu_+-\nu+\hbar)}\,X_i^-(\nu)\,, \qquad i < M+1, \\
\Psi_i^-(\mu)^{-1}\,X_i^-(\nu)\,\Psi_i^-(\mu) &= \frac{(\mu_+-\nu+\hbar)}{(\mu_+-\nu-3\hbar)}\,X_i^-(\nu)\,, \qquad M+N+1 \geq i > M+1, \\
\Psi_{i+1}^+(\mu)^{-1}\,X_i^-(\nu)\,\Psi_{i+1}^+(\mu) &= \frac{(\mu_--\nu+2\hbar)}{(\mu_--\nu)}\,X_i^-(\nu) \quad i < M+1, \\
\Psi_{i+1}^+(\mu)^{-1}\,X_i^-(\nu)\,\Psi_{i+1}^+(\mu) &= \frac{(\mu_--\nu)}{(\mu_--\nu+2\hbar)}\,X_i^-(\nu)\,, \qquad M+N+1 \geq i \geq M+1, \\
\Psi_{i+1}^-(\mu)^{-1}\,X_i^-(\nu)\,\Psi_{i+1}^-(\mu) &= \frac{(\mu_+-\nu)}{(\mu_+-\nu-2\hbar)}\,X_i^-(\nu)\,, \qquad i < M+1,
\end{aligned}
$$

$$
\begin{aligned}
\Psi_{i+1}^{-}(\mu)^{-1} X_i^{-}(\nu) \Psi_{i+1}^{-}(\mu) &= \frac{(\mu_+ - \nu - 2\hbar)}{(\mu_+ - \nu)} X_i^{-}(\nu), \quad M+N+1 \geq i \geq M+1, \\
\Psi_{M+1}^{\pm}(\mu)^{-1} X_{M+1}^{\pm}(\nu) \Psi_{M+1}^{\pm}(\mu) &= X_{M+1}^{\pm}(\nu) \\
\Psi_{M+1}^{\pm}(\mu)^{-1} X_{M+1}^{\mp}(\nu) \Psi_{M+1}^{\pm}(\mu) &= X_{M+1}^{\mp}(\nu) \\
(\mu - \nu \mp 2\hbar) X_i^{\mp}(\mu) X_i^{\mp}(\nu) &= (\mu - \nu \pm 2\hbar) X_i^{\mp}(\nu) X_i^{\mp}(\mu), \quad i < M+1, \\
(\mu - \nu \pm 2\hbar) X_i^{\mp}(\mu) X_i^{\mp}(\nu) &= (\mu - \nu \mp 2\hbar) X_i^{\mp}(\nu) X_i^{\mp}(\mu), \quad M+N+1 \geq i > M+1, \\
(\mu - \nu \pm \hbar) X_i^{\mp}(\mu) X_{i+1}^{\mp}(\nu) &= (\mu - \nu \mp \hbar) X_{i+1}^{\mp}(\nu) X_i^{\mp}(\mu), \quad i < M+1, \\
(\mu - \nu \mp \hbar) X_i^{\mp}(\mu) X_{i+1}^{\mp}(\nu) &= (\mu - \nu \pm \hbar) X_{i+1}^{\mp}(\nu) X_i^{\mp}(\mu), \quad M+N+1 \geq i \geq M+1, \\
\left[X_i^{+}(\mu), X_j^{-}(\nu)\right] &= -2\hbar\delta_{ij} \left(\delta(\mu_- - \nu_+) \Psi_i^{-}(\nu_+) - \delta(\mu_+ - \nu_-) \Psi_i^{+}(\mu_+)\right), \quad i, j \neq M+1, \\
\left\{X_{M+1}^{+}(\mu) \ , \ X_{M+1}^{-}(\nu)\right\} &= 2\hbar \left(\delta(\mu_- - \nu_+) \Psi_i^{-}(\nu_+) - \delta(\mu_+ - \nu_-) \Psi_i^{+}(\mu_+)\right),
\end{aligned}
$$

where [X,Y]$\equiv XY - YX$ stands for a commutator and {X, Y}$\equiv XY + YX$ for an anti-commutator. As for the serre relations for the Drinfeld currents, we will refer the reader to the ref.[19].

We should remark that our definition of $DY_\hbar\left(\widehat{sl}(M+1|N+1)\right)$ is equivalent to that of Zhang's[19] with defining

$$
\Psi_j^{\pm}(\mu) = k_{j+1}^{\mp}(\mu) k_j^{\mp}(\mu)^{-1},
$$

and spectra-shifting

$$
\begin{aligned}
\Psi_j^{\pm}(\mu) &\rightarrow \Psi_j^{\pm}(\mu + (i-1)\hbar) \quad i < M+1, \\
\Psi_j^{\pm}(\mu) &\rightarrow \Psi_j^{\pm}(\mu + (M-i)\hbar) \quad M+N+1 \geq i \geq M+1. \\
X_j^{\pm}(\mu) &\rightarrow X_j^{\pm}(\mu + (i-1)\hbar) \quad i < M+1, \\
X_j^{\pm}(\mu) &\rightarrow X_j^{\pm}(\mu + (M-i)\hbar) \quad M+N+1 \geq i \geq M+1.
\end{aligned}
$$

3 Free Bosonic realization

Now, we study the free bosonic realization of $DY_\hbar\left(\widehat{sl}(M+1|N+1)\right)$ which give us a level one representation. Free bosonic realizations of the level-one representations of quantum affine super algberas have been obtained for $U_q[\widehat{sl}(M+1|N+1)]$, $M \neq N$[10] and $U_q[\widehat{gl}(N|N)]$[11]. Our representation can be regarded as extending the above works to the super-affine-Yangian case.

Let introduce bosonic oscillators $\{\widehat{a}_i(t), \widehat{b}_j(t), \widehat{c}_j(t), |i = 1, \cdots, M+1, j = 1, \cdots, N+1\}$ with a continuous parameters $t(t \in R - \{0\}$, which satisfy the following commutation relations

$$\begin{aligned} \left[\widehat{a}_i\left(t\right) \widehat{a_j}\left(t'\right)\right] &= \frac{1}{\left(i\hbar\right)^2 t} sh^2\left(i\hbar t\right) \delta_{ij} \delta\left(t+t'\right), \qquad (3.1) \\ \left[\widehat{b}_i\left(t\right) \widehat{b}_j\left(t'\right)\right] &= -\frac{1}{\left(i\hbar\right)^2 t} sh^2\left(i\hbar t\right) \delta_{ij} \delta\left(t+t'\right), \qquad (3.2) \\ \left[\widehat{c}_i\left(t\right) \widehat{c}_j\left(t'\right)\right] &= \frac{1}{\left(i\hbar\right)^2 t} sh^2\left(i\hbar t\right) \delta_{ij} \delta\left(t+t'\right), \qquad (3.3) \end{aligned}$$

and the other commutators vanish. Define bosonic oscillators $\widehat{\lambda_i}(t)(1 \le i \le M+N+1)$

$$\begin{aligned} \widehat{\lambda}_i(t) &= \widehat{a}_i(t)e^{i\hbar\frac{|t|}{2}} - \widehat{a}_{i+1}(t)e^{-i\hbar\frac{|t|}{2}}, \quad i = 1, \cdots, M, \\ \widehat{\lambda}_{M+1}(t) &= \widehat{a}_{M+1}(t)e^{i\hbar\frac{|t|}{2}} + \widehat{b}_1(t)e^{i\hbar\frac{|t|}{2}}, \\ \widehat{\lambda}_{M+1+j}(t) &= -\widehat{b_j}(t)e^{-i\hbar\frac{|t|}{2}} + \widehat{b}_{j+1}(t)e^{i\hbar\frac{|t|}{2}}, \quad j = 1, \cdots, N. \end{aligned}$$

It is easy to verify that the bosonic oscillators satisfy

$$\left[\widehat{\lambda_i}\left(t\right) \widehat{\lambda_j}\left(t'\right)\right] = \frac{1}{\left(i\hbar\right)^2 t} sh\left(ia_{ij}\hbar t\right) sh\left(i\hbar t\right) \delta\left(t+t'\right), \qquad (3.4)$$

where a_{ij} is the Cartan matrix of the super algebra $sl(M+1|N+1)$

$$(a_{ij}) = \begin{pmatrix} 2 & -1 & & & & & & & \\ -1 & 2 & \ddots & & & & & & \\ & \ddots & \ddots & -1 & & & & & \\ & & -1 & 2 & -1 & & & & \\ & & & -1 & 0 & 1 & & & \\ & & & & 1 & -2 & \ddots & & \\ & & & & & \ddots & \ddots & 1 & \\ & & & & & & 1 & -2 & 1 \\ & & & & & & & 1 & -2 \end{pmatrix} \quad (1 \le i, j \le M+N+1). \qquad (3.5)$$

Moreover, we introduce the zero mode operators $\{a_i(0), b_j(0), c_j(0), Q_{a_i}, Q_{b_j}, Q_{c_j} | i = 1, \cdots, M+1, j = 1, \cdots, N+1\}$ satisfying the following commutation relations

$$\begin{aligned} [a_i(0), Q_{a_i}] &= \delta_{ij}, \qquad (3.6) \\ [b_i(0), Q_{b_i}] &= -\delta_{ij}, \qquad (3.7) \\ [c_i(0), Q_{c_i}] &= \delta_{ij}. \qquad (3.8) \end{aligned}$$

Define $Q_{\widehat{\lambda}_i} = Q_{a_i} - Q_{a_{i+1}}$ for $i = 1, \cdots, M$, $Q_{\widehat{\lambda}_{M+1}} = Q_{a_{M+1}} + Q_{b_1}$ and $Q_{\widehat{\lambda}_{M+1+j}} = -Q_{b_j} + Q_{b_{i+1}}$ for $j = 1, \cdots, N$. Let us introduce the notations

$$\begin{aligned} \chi_i(\mu;\beta) &= \exp\left\{-i\hbar\int_{-\infty}^{0}\frac{dt}{sh\,(i\hbar t)}\widehat{\chi_i}\,(t)\,e^{-i\hbar\beta t}e^{-i\mu t}\right\} \\ &\quad\times\exp\left\{-i\hbar\int_{-\infty}^{0}\frac{dt}{sh\,(i\hbar t)}\widehat{\chi_i}\,(t)\,e^{-i\hbar\beta t}e^{-i\mu t}\right\}e^{Q_{\widehat{\chi_i}}}, \qquad (3.9) \\ \chi_i^{+}\,(\mu) &= \exp\left\{-2\hbar\int_{0}^{\infty}\widehat{\chi_i}\,(t)\,e^{-i\mu t}dt\right\}, \qquad (3.10) \\ \chi_i^{-}\,(\mu) &= \exp\left\{2\hbar\int_{-\infty}^{0}\widehat{\chi_i}\,(t)\,e^{-i\mu t}dt\right\}. \qquad (3.11) \end{aligned}$$

In the following, we will adopt these notations for the other bosonic fields $\{\lambda_i(\mu;\beta), c_j(\mu;\beta)$ $|i = 1, \cdots, M + N + 1,\ j = 1, \cdots, N + 1\}$, for example, the bosonic field $c_j(\mu;\beta)$ should be defined in the same way. We introduce the $\hbar$-difference operator defined by

$$D_{\hbar}\,(f(\beta)) = f(\beta - i\hbar) - f(\beta + i\hbar). \qquad (3.12)$$

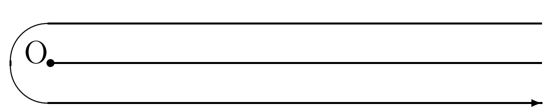
When calculating the normal order of the bosonic fields, one often encounter an integral

$$\int_0^{\infty} F(t)dt,$$

which is divergent at t=0 . Here we adopt the following regularization[17, 16]. The integral should be understood as the contour integral

$$\int_C F(t)\frac{\ln(-t)}{2\pi i}dt,$$

where C indicate the circle from $+\infty$ to 0 in the upper half plain and 0 to ∞ in the lower half plain



From the above regularization and after a straightforward calculation, we can obtain the normal order relations of the basic bosonic fields which will be given in the appendix A. Using the normal order relations, we can construct the level-one representation of

$DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ in terms of free bosonic fields. Namely, the Drinfeld currents of $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ at level-one are realized by the free boson fields as follow

$$X_i^+(\mu) = :\lambda_i\left(\mu;\frac{1}{2}\right)D_\hbar\left[c_{i-M-1}^{-1}(\mu;0)\right]c_{i-M}(\mu;0):,\quad M+N+1\geq i>M+1, \tag{3.13}$$

$$X_i^+(\mu) = \frac{\sqrt{2\hbar}}{\exp\left(-\frac{1}{2}\gamma\right)}:\lambda_i\left(\mu;\frac{1}{2}\right)c_{i-M}(\mu;0):\prod_{j=1}^{M}\exp\left(-i\pi a_j(0)\right),\quad i=M+1, \tag{3.14}$$

$$X_i^+(\mu) = \frac{2\hbar}{\exp(-\gamma)}:\lambda_i\left(\mu;\frac{1}{2}\right):\exp\left(i\pi a_i(0)\right),\quad i<M+1, \tag{3.15}$$

$$X_i^-(\mu) = :\lambda_i^{-1}\left(\mu;-\frac{1}{2}\right)c_{i-M-1}^{-1}(\mu;0)\,D_\hbar[c_{i-M}^{-1}(\mu;0)]:\quad M+N+1\geq i>M+1, \tag{3.16}$$

$$X_i^-(\mu) = \frac{\sqrt{2\hbar}}{\exp\left(-\frac{1}{2}\gamma\right)}:\lambda_i^{-1}\left(\mu;-\frac{1}{2}\right)D_\hbar[c_{i-M}^{-1}(\mu;0)]:\prod_{j=1}^{M}\exp\left(i\pi a_j(0)\right)\quad i=M+1, \tag{3.17}$$

$$X_i^-(\mu) = \frac{2\hbar}{\exp(-\gamma)}:\lambda_i^{-1}\left(\mu;-\frac{1}{2}\right):\exp\left(-i\pi a_i(0)\right)\quad i<M+1, \tag{3.18}$$

$$\Psi_i^\pm(\mu) = \lambda_i^\pm(\mu). \tag{3.19}$$

This bosonic realization may help us to construct the bosonization of the supersymmetric $t-J$ model.

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Appendix A

In this appendix, we give the normal order relations of the fundamental bosonic fields. In order to calculate the normal order relations, the following formula[16] is helpful

$$\int_C \frac{d\lambda \ln(-\lambda)}{2\pi i\lambda}\frac{e^{-x\lambda}}{1-e^{-\lambda/\eta}} = \ln\Gamma(\eta x) + (\eta x - \frac{1}{2})(\gamma - \ln\eta) - \frac{1}{2}\ln 2\pi$$

where γ is the Euler's constant. Using the integral representation of Γ -functions and the definition of Drinfeld currents of $DY_\hbar\left(\widehat{sl}\left(M+1|N+1\right)\right)$ at level-one, we can derive

$$\begin{aligned}
\lambda_i(\mu;\beta_1)\lambda_i(\nu;\beta_2) &= \frac{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)-\frac{(\beta_1+\beta_2)}{2}+\frac{(1+a_{ii})}{2}\right]}{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)-\frac{(\beta_1+\beta_2)}{2}+\frac{(1-a_{ii})}{2}\right]}\frac{e^{a_{ii}\gamma}}{\eta^{a_{ii}}} : \lambda_i(\mu;\beta_1)\lambda_i(\nu;\beta_2) :, \\
\lambda_i(\mu;\beta_1)\lambda_{i+1}(\nu;\beta_2) &= \frac{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)-\frac{(\beta_1+\beta_2)}{2}+\frac{(1+a_{i,i+1})}{2}\right]}{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)-\frac{(\beta_1+\beta_2)}{2}+\frac{(1-a_{i,i+1})}{2}\right]}\frac{e^{a_{i,i+1}\gamma}}{\eta^{a_{i,i+1}}} : \lambda_i\left(\mu;\beta_1\right)\lambda_{i+1}\left(\nu;\beta_2\right) :, \\
\lambda_i(\mu;\beta_1)\lambda_j(\nu;\beta_2) &= : \lambda_i(\mu;\beta_1)\lambda_j(\nu;\beta_2) :, \quad |i-j|>1, \\
c_i(\mu;0)c_j(\nu;0) &= \delta_{ij}\,\frac{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)+1\right]}{\Gamma\left[\frac{1}{2\hbar}\left(\mu-\nu\right)\right]}\,\frac{e^{\gamma}}{\eta} : c_i(\mu;0)c_j(\nu;0) :,
\end{aligned}$$

where $\eta = \frac{1}{2\hbar}$ and a_{ij} is the Cartan matrix (3.5)

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